Published by Institute of Physics Publishing for SISSA
Received: *December 14, 2006*
Revised: *January 25, 2007*
Accepted: *January 25, 2007*
Published: *February 20, 2007*

Hunting η_b through radiative decay into J/ψ

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Abstract: We propose that the radiative decay process, $\eta_b \to J/\psi\,\gamma$, may serve as a clean searching mode for η_b in hadron collision facilities. By a perturbative QCD calculation, we estimate the corresponding branching ratio to be of order 10^{-7}. Though very suppressed, this radiative decay channel in fact has larger branching ratio than the hadronic decay process $\eta_b \to J/\psi\,J/\psi$, which was previously hoped to be a viable mode to search for η_b in Tevatron Run 2. The discovery potential of η_b through this channel seems promising in the forthcoming LHC experiments, and maybe even in Tevatron Run 2, thanks to the huge statistics of η_b to be accumulated in these experiments. The same calculational scheme is also used to estimate the branching ratios for the processes $\eta_b\,(\eta_c) \to \phi\gamma$.

Keywords: Hadronic Colliders, QCD, Electromagnetic Processes and Properties, Heavy Quark Physics.

Contents

1. Introduction

The existence of η_b, the pseudo-scalar partner of $\Upsilon(1S)$, is a firm prediction of QCD, about which nobody would seriously challenge. It is rather unsatisfactory that although three decades have elapsed since the discovery of $\Upsilon(1S)$, this particle eludes intensive experimental endeavors and still eagerly awaits to be established.

On the theoretical side, many work have been devoted to uncovering various properties of η_b, such as its mass, inclusive hadronic and electromagnetic widths, transition rates and production cross sections in different collider programs [1]. Among all the observables, its mass is believed to be the simplest and most unambiguous to predict. Recent estimates for $\Upsilon - \eta_b$ mass splitting span the 40–60 MeV range [2 − 5]. An eventual definite sighting of η_b and precise measurement of its mass will critically differentiate varieties of theoretical approaches, consequently sharpening our understanding towards the $b\bar{b}$ ground state.

The η_b has recently been sought from $\gamma\gamma$ collisions in the full LEP 2 data sample, where approximately few hundreds of η_b are expected to be produced. ALEPH has one candidate event with the reconstructed mass of 9.30 ± 0.03 GeV, but consistent to be a background event [6]. ALEPH, L3, DELPHI have also set upper limits on the branching fractions for η_b decays into 4, 6, 8 charged particles [6 − 8]. Based on the 2.4 fb^{-1} data taken at the $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances, CLEO has searched distinctive single photons from hindered $M1$ transitions $\Upsilon(2S), \Upsilon(3S) \to \eta_b\gamma$, and from the cascade decay $\Upsilon(3S) \to h_b\pi^0$, $h_b\pi^+\pi^-$ followed by $E1$ transition $h_b \to \eta_b\gamma$, but no signals have been found [9].

Hadron collider experiments provide an alternative means to search for η_b. Unlike the e^+e^- machines which are limited by the low yield of η_b, hadron colliders generally possess a much larger η_b production rate, which in turn allows for triggering it through some relatively rare decay modes yet with clean signature. However, one caveat is also worth being emphasized. The corresponding background events are in general copious in hadron machines, so the virtue of this sort of decay modes may be seriously discounted (Such an example is the electromagnetic decay $\eta_b \to \gamma\gamma$, with an expected branching fraction $\sim 10^{-4}$, which is nevertheless overshadowed by the ubiquitous γ events originating from π^0 decay).

Several years ago, Braaten, Fleming and Leibovich suggested that the hadronic decay $\eta_b \to J/\psi J/\psi$, followed by both J/ψ decays to muon pairs, can be used as a very clean trigger to search for η_b at Tevatron Run 2 [10]. By some simple scaling assumption, they estimate the branching ratio of the double J/ψ mode to be $7 \times 10^{-4\pm1}$, and conclude that the prospect of observing the $\eta_b \to 4\mu$ channel at Tevatron Run 2 is rather promising. CDF in fact has followed this suggestion and looked at the full Run 1 data for the 4μ events in the expected η_b mass window [11].

However, some objection has been recently put forward by Maltoni and Polosa, who argue that the estimate of the branching ratio for double J/ψ mode by Braaten *et al* might be too optimistic [12]. They instead advocate that $\eta_b \to D^*\overline{D}^{(*)}$, with the estimated decay ratios lying in the $10^{-3} - 10^{-2}$ range, may serve as better searching modes for η_b in Run 2.

Very recently, one of us (Y. J.) has surveyed the discovery potential of various hadronic decay channels of η_b [13]. The explicit perturbative QCD (pQCD) calculation predicts $\mathrm{Br}[\eta_b \to J/\psi J/\psi]$ to be only of order 10^{-8}. If this is the case, the chance of observing this decay mode in Run 2 then becomes rather gloomy, whereas the observation possibility at LHC may still remain. Another noteworthy assertion of [13] is that, by some rough but physical considerations, one expects $\mathrm{Br}[\eta_b \to D^*\overline{D}] \sim 10^{-5}$ and $\mathrm{Br}[\eta_b \to D^*\overline{D}^*] \sim 10^{-8}$, which are much smaller than the optimistic estimates by Maltoni and Polosa. Taking the low reconstruction efficiency of D mesons further into consideration, these double charmful decay modes may not be as attractive as naively thought.

In this paper, we try to propose another decay process, $\eta_b \to J/\psi\gamma$, as a viable discovery channel for η_b in hadron collider experiments. At first sight, this mode, being a radiative decay process, may not look very economical due to the large total width of η_b. In fact, our explicit pQCD calculation reveals that the corresponding branching ratio is indeed very suppressed, only about 10^{-7}. Nevertheless, it is worth emphasizing that this is already *larger* than that of the double J/ψ mode. The usual way of reconstructing J/ψ is via the dimuon mode, with a branching fraction about 6%. In the light of this, the advantage of this radiative decay process over the double J/ψ mode will be further amplified, since only one J/ψ needs to be reconstructed in our case. Unlike the open-charm decay processes $\eta_b \to D^*\overline{D}^{(*)}$, in which the final decay products such as K, π in general suffer severe contamination from the combinatorial background, the presence of J/ψ in the final state renders our radiative decay channel much cleaner to look for experimentally.

Thanks to the huge amount of η_b to be accumulated in high energy hadron collision facilities, we expect that this radiative decay channel, albeit being a rare decay mode, may still have bright prospect to be observed in Run 2 and in the forthcoming LHC program. Not surprisingly, one should be aware that the major QCD background events, i.e. the associated $J/\psi + \gamma$ production [14–16], may exceedingly outnumber our signal events. To

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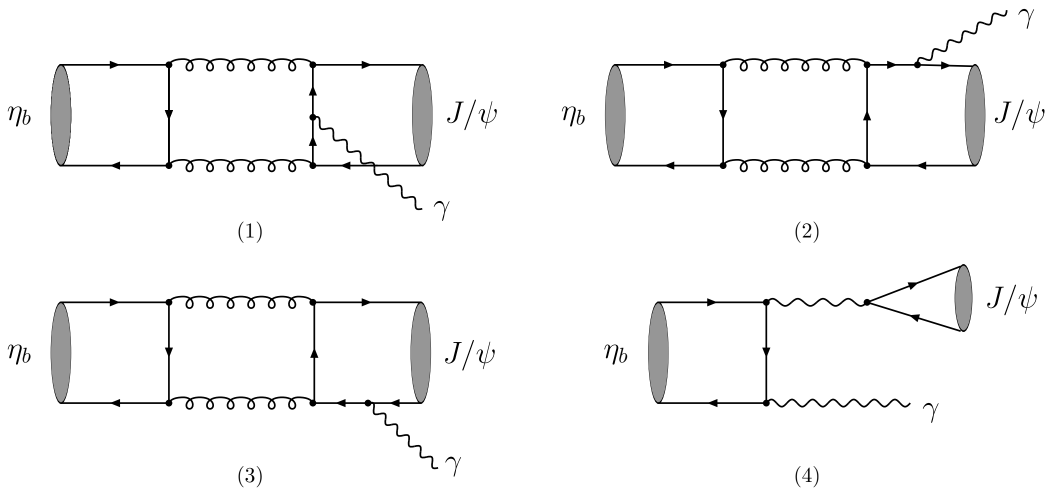

Figure 1: Lowest-order diagrams that contribute to $\eta_b \to J/\psi\,\gamma$. Diagrams (1), (2), (3) stand for QCD-initiated process, while (4) represents the dominant QED process governed by photon fragmentation. Crossed diagrams are implicitly implied.

make this mode practically viable, one has to ensure that those background events can be significantly depressed by judiciously adjusting the kinematical cuts.

The remainder of the paper is distributed as follows. In section 2, we present the pQCD calculation for the radiative decay process $\eta_b \to J/\psi\gamma$, treating heavy quarkonium states in NRQCD approach which, to our purpose, is equivalent to the color-singlet model. In section 3, we present the numerical prediction to the corresponding branching ratio, and explore the observation potential of this decay mode in Tevatron Run 2 and in the coming LHC experiment. We subsequently apply the same formalism to estimate the branching ratios for analogous processes $\eta_b\,(\eta_c) \to \phi\gamma$, which is equivalent to work in the context of constituent quark model. We summarize in section 4.

2. Color-singlet model calculation

In this section, we present a pQCD calculation for the decay rate of $\eta_b \to J/\psi\gamma$. This decay process can be initiated by either strong or electromagnetic interaction, with the corresponding lowest-order diagrams shown in figure 1. Since the annihilation of the $b\bar{b}$ pair, the creation of the $c\bar{c}$ pair, as well as the emission of the hard photon, are all dictated by short-distance physics, with the hard scales set by the heavy quark masses, it is appropriate to utilize the pQCD scheme to tackle this exclusive process.

Nowadays it has become standard to employ the NRQCD factorization framework to cope with hard processes involving heavy quarkonia [17]. One of the major advantages of this model-independent EFT approach over the old color-singlet model is that it can in principle incorporate the contribution of higher Fock components of quarkonium (*color-octet effect*) in a systematic fashion. This is exemplified by the systematic treatment of inclusive annihilation decay of various quarkonium states within this framework [17, 18]. In contrast, the inclusion of color-octet effect in exclusive processes has not been developed to a comparable level within this context. To date, there are only few cases where this effect has been investigated in the model-independent language. One example is the magnetic dipole transition process such as $J/\psi \to \eta_c\gamma$, where the leading color-octet contribution vanishes [19]. Another example is the quarkonium radiative decay to light mesons, such as $\Upsilon \to f_2(1270)\gamma$, where the color-octet effect turns out to be quite insignificant [20].

If we take the lesson from the aforementioned radiative decay processes, it seems persuasive to assume that the color-octet effect may also be unimportant in our case. We will completely ignore this effect in this work. In this regard, there will be no difference between the NRQCD approach and the color-singlet model, consequently these two terms will be used interchangeably.

Before launching into the actual calculation, it is worth mentioning that, Guberina and Kuhn have studied the analogous radiative decay process $\Upsilon \to \eta_c\gamma$ in NRQCD approach more than two decades ago [21]. Our calculation will intimately resemble theirs.

It is useful to take notice of a simple trait of this process, that the photon can only be transversely polarized, so is the recoiling J/ψ by angular momentum conservation. This property holds true irrespective of whether this process is initiated by strong or electromagnetic interaction. In fact, parity and Lorentz invariance constrains the decay amplitude to have the following unique tensor structure:

$$\mathcal{M}(\lambda_1,\lambda_2) = \mathcal{A}\,\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\mu}_{J/\psi}(\lambda_1)\,\varepsilon^{*\nu}_{\gamma}(\lambda_2)\,Q^\alpha\,k^\beta\,. \tag{2.1}$$

We use Q, P and k to signify the momenta of η_b, J/ψ and γ, respectively, and use λ_1 and λ_2 to label the helicities of J/ψ and γ which are viewed in the η_b rest frame. It is straightforward to infer from (2.1), that the only physically allowed helicity configurations are $(\lambda_1,\ \lambda_2)$=$(\pm1,\ \pm1)$. All the dynamics is encoded in the coefficient $\mathcal{A}$, which we call *reduced* amplitude. Our task then is to find out its explicit form.

In the color-singlet-model approach, it is customary to start with the parton process $b(p_b)\,\bar{b}(p_{\bar{b}}) \to c(p_c)\,\bar{c}(p_{\bar{c}}) + \gamma(k)$, then project this matrix element onto the corresponding color-singlet quarkonium Fock states. For reactions involving heavy quarkonium, it is conventional to organize the amplitude in powers of relative momentum between its constituents, to accommodate the NRQCD ansatz. This work is intended only for the leading order accuracy in relativistic expansion, it is then legitimate to neglect the relative momentum inside both η_b and J/ψ. We thus set $p_b = p_{\bar{b}} = Q/2$ and $p_c = p_{\bar{c}} = P/2$. For the $b\bar{b}$ pair to form η_b, it is necessarily in a spin-singlet and color-singlet state, and one can replace the product of the Dirac and color spinors for b and $\bar{b}$ in the initial state with the projector

$$u(p_b)\,\overline{v}(p_{\bar{b}}) \longrightarrow \frac{1}{2\sqrt{2}}\,(\not{Q} + 2m_b)\,i\gamma_5 \times \left(\frac{1}{\sqrt{m_b}}\psi_{\eta_b}(0)\right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}}\right)\,. \tag{2.2}$$

For the outgoing J/ψ, one can employ the following projection operator:

$$v(p_{\bar{c}})\,\overline{u}(p_c) \longrightarrow \frac{1}{2\sqrt{2}}\,\not{\varepsilon}^{*}_{J/\psi}\,(\not{P} + 2m_c)\,\times \left(\frac{1}{\sqrt{m_c}}\psi_{J/\psi}(0)\right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}}\right)\,, \tag{2.3}$$

where $\varepsilon^{\mu}_{J/\psi}$ is the polarization vector of J/ψ satisfying $\varepsilon_{J/\psi}(\lambda)\cdot\varepsilon^{*}_{J/\psi}(\lambda') = -\delta^{\lambda\lambda'}$ and $P\cdot\varepsilon = 0$. $N_c = 3$, and $\mathbf{1}_c$ stands for the unit color matrix. The nonperturbative parameters, $\psi_{\eta_b}(0)$ and $\psi_{J/\psi}(0)$, are Schrödinger wave functions at the origin for η_b and J/ψ, which can be either inferred from phenomenological potential models or (in)directly extracted from experiments. By writing (2.2) and (2.3) the way as they are, it is understood that $M_{\eta_b} = 2m_b$ and $M_{J/\psi} = 2m_c$ have been assumed.

We commence with the strong decay amplitude. As can be seen in figure 1, the lowest order QCD-initiated contribution starts already at one loop. Note that the photon can only be emitted form the c quark line, because the even C-parity of η_b forbids the γ to attach to its constituents. Using the projection operators in (2.2) and (2.3), it is straightforward to write down the Feynman rules for the strong decay amplitude:

$$\begin{aligned}
\mathcal{M}_{\rm str} \,=\, & -C_{\rm str}\, e_c e\, g_s^4\, \frac{\psi_{\eta_b}(0)\,\psi_{J/\psi}(0)}{8\sqrt{m_b m_c}} \int \frac{d^4k_1}{(2\pi)^4}\,\frac{1}{k_1^2}\,\frac{1}{k_2^2} \\
& \times \left[\frac{{\rm tr}[(\not{Q}+2m_b)\gamma_5\gamma_\nu(\not{p}_b-\not{k}_1+m_b)\gamma_\mu]}{(p_b-k_1)^2-m_b^2} + (\mu\leftrightarrow\nu,\ k_1\leftrightarrow k_2)\right] \\
& \times \left\{ \frac{{\rm tr}[\not{\varepsilon}^{*}_{J/\psi}(\not{P}+2m_c)\gamma^\mu(\not{p}_c-\not{k}_1+m_c)\,\not{\varepsilon}^{*}_{\gamma}(\not{k}_2-\not{p}_{\bar c}+m_c)\gamma^\nu]}{((p_c-k_1)^2-m_c^2)\,((p_{\bar c}-k_2)^2-m_c^2)} \right. \\
& + \frac{{\rm tr}[\not{\varepsilon}^{*}_{J/\psi}(\not{P}+2m_c)\,\not{\varepsilon}^{*}_{\gamma}(\not{p}_c+\not{k}+m_c)\gamma^\mu(\not{k}_2-\not{p}_{\bar c}+m_c)\gamma^\nu]}{((p_c+k)^2-m_c^2)\,((p_{\bar c}-k_2)^2-m_c^2)} \\
& \left. + \frac{{\rm tr}[\not{\varepsilon}^{*}_{J/\psi}(\not{P}+2m_c)\gamma^\mu(\not{p}_c-\not{k}_1+m_c)\gamma^\nu(-\not{p}_{\bar c}-\not{k}+m_c)\,\not{\varepsilon}^{*}_{\gamma}]}{((p_c-k_1)^2-m_c^2)\,((p_{\bar c}+k)^2-m_c^2)} \right\}, \qquad (2.4)
\end{aligned}$$

where the corresponding color factor $C_{\rm str} = N_c^{-1}\,{\rm tr}(T^aT^b){\rm tr}(T^aT^b) = \frac{2}{3}$. The momenta carried by two internal gluons are labeled by k_1, k_2, respectively, which are subject to the constraint $k_1 + k_2 = Q$. Notice that the box diagrams in figure 1 are related to one-loop four- or five-point functions, and the corresponding loop integrals are ultraviolet finite. Moreover, the occurrence of heavy b and c masses ameliorate the infrared behavior of the loop integral so that the result turns out to be simultaneously infrared finite. Since there is no need for regularization, we have directly put the spacetime dimension to four.

After completing the Dirac trace in (2.4), we end up with terms which do not immediately possess the desired tensor structure of (2.1), instead with one index of Levi-Civita tensor contracted to the loop momentum variable. Of course, when everything is finally worked out, all these terms must conspire to arrive at the desired Lorentz structure. Conversely, one may exploit this knowledge to simplify the algebra prior to performing the loop integral [21, 22]. First pull out the partial amplitude $M_{\mu\nu}$ through $\mathcal{M}_{\rm str} = M_{\mu\nu}\,\varepsilon^{*\mu}_{J/\psi}\,\varepsilon^{*\nu}_{\gamma}$. Eq. (2.1) then demands

$$M_{\mu\nu} = \mathcal{A}_{\rm str}\,\epsilon_{\mu\nu\alpha\beta}\,Q^\alpha\,k^\beta\,, \qquad (2.5)$$

which is compatible with the expectation that $M_{\mu\nu}$ would vanish unless μ, ν are in transverse directions. Contracting both sides of (2.5) with $\epsilon^{\mu\nu\rho\sigma}Q_\rho k_\sigma$, we can express the reduced amplitude as

$$\mathcal{A}_{\rm str} = \frac{1}{2\,(k\cdot Q)^2}\, M_{\mu\nu}\,\epsilon^{\mu\nu\rho\sigma}\,Q_\rho\,k_\sigma\,. \tag{2.6}$$

After this manipulation is done, it is convenient to adopt a new loop momentum variable q, which is related to the old one via $k_1 = (Q+q)/2$ and $k_2 = (Q-q)/2$. We end in a concise expression:

$$\mathcal{A}_{\rm str} = -\frac{e_c e\, g_s^4}{6\,\pi^2}\sqrt{\frac{m_c}{m_b}}\,\frac{\psi_{\eta_b}(0)\,\psi_{J/\psi}(0)}{(m_b^2-m_c^2)^2}\, f\left(\frac{m_c^2}{m_b^2}\right), \tag{2.7}$$

where

$$f\left(\frac{m_c^2}{m_b^2}\right) = \frac{8}{i\pi^2}\int d^4q\, \frac{(k\cdot Q)\,q^2 - (k\cdot q)\,(Q\cdot q)}{(q+Q)^2\,(q-Q)^2\,(q^2+2k\cdot q-P^2)\,(q^2-2k\cdot q-P^2)}\,. \tag{2.8}$$

Since f is dimensionless, it can depend upon m_b and m_c only through their dimensionless combination. It is interesting to note that the b quark propagator has been canceled in this expression. The loop integral can be performed analytically by the standard method, and the result is

$$\begin{aligned}
{\rm Re}f(u) =&\ \frac{2(1-u)}{2-u}\ln\left[\frac{u}{2(1-u)}\right] - \frac{2}{1+u}\left\{\ln^2 2 + \frac{1}{2}\ln^2 u + \ln[2-u]\ln\left[\frac{u}{2(1-u)}\right]\right.\\
&+\ln u \ln\left[\frac{2}{1-u}\right] - u\ln\left[\frac{u}{2-u}\right]\ln\left[\frac{u}{2(1-u)}\right] + 2\,{\rm Li}_2[-u] + {\rm Li}_2\left[\frac{u-1}{2u}\right]\\
&\left.+ 2\,{\rm Li}_2\left[\frac{u}{2}\right] - {\rm Li}_2\left[\frac{u^2-u}{2}\right] - u\,{\rm Li}_2\left[2-\frac{2}{u}\right]\right\}, \qquad (2.9)\\
{\rm Im}f(u) =&\ 2\pi\left\{\frac{1-u}{2-u} + \frac{u\ln u}{1+u} - \ln[2-u]\right\}, \qquad (2.10)
\end{aligned}$$

where ${\rm Li}_2$ denotes dilogarithm (Spence) function. We will expound the derivation of this result in the appendix. Note f has an absorptive part, which reflects that the intermediate gluons are kinematically permissible to stay on shell. One can apply Cutkosky's cutting rule to verify (2.10). The real and imaginary parts of f as function of m_c^2/m_b^2 are displayed in figure 2.

It is instructive to know the asymptotic behavior of f in the $u\to 0$ limit, which can be readily read out from (2.9) and (2.10):

$$\begin{aligned}
{\rm Re}f(u) &\longrightarrow (1-2\ln 2)\ln u + \frac{\pi^2}{3} - \ln 2 + \ln^2 2\,, &(2.11)\\
{\rm Im}f(u) &\longrightarrow \pi\,(1-2\ln 2)\,. &(2.12)
\end{aligned}$$

Note as u approaches 0, the imaginary part remains finite, whereas the real part blows up logarithmically. This trend can be clearly visualized in figure 2. The logarithmic divergence in the $m_c\to 0$ limit is obviously of infrared origin. Nevertheless, this does not pose any practical problem, since a non-relativistic description for a zero-mass bound state, as well as the resulting predictions, should not be trusted anyway. It is interesting to note that,

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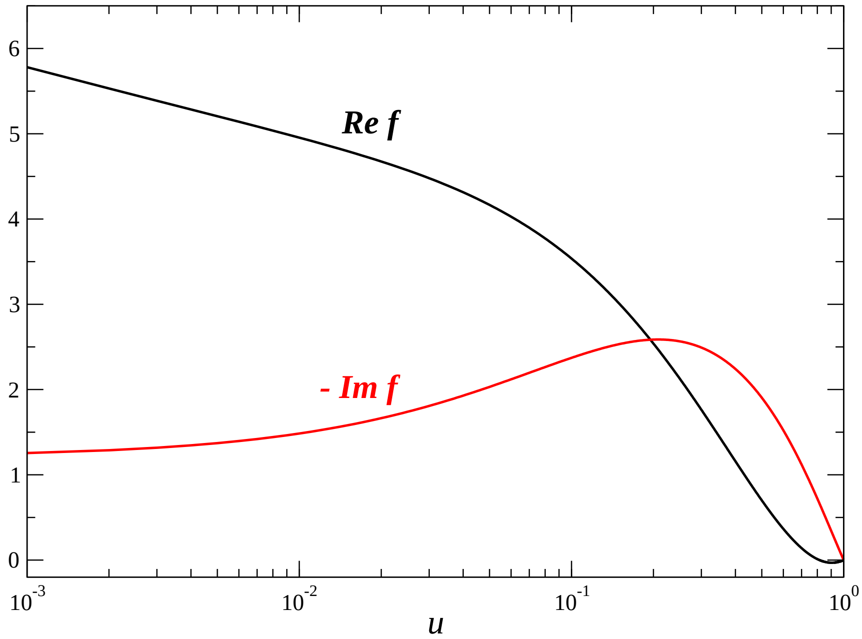

Figure 2: Real and imaginary parts of $f(u)$.

the analogous radiative decay process, $\Upsilon \to \eta_c\gamma$, has a different asymptotic behavior, in which both the real and imaginary parts of the QCD amplitude admits a finite limit [21].

We next turn to the pure QED contribution to this radiative decay process. One may naively expect that this contribution is much less important than the QCD contribution. However, it turns out that this expectation is not true and the QED contribution must be retained. Due to the neutral color charge of photon, this radiative decay can arise at tree level from the photon fragmentation, as shown in figure 1 (4). Obviously, the fragmentation type contribution is much more dominant over other types of QED diagrams. To calculate this contribution, a necessary input is the $\gamma - J/\psi$ coupling, which is characterized by the J/ψ decay constant:[1]

$$\langle J/\psi(\lambda)|\bar{c}\gamma^\mu c|0\rangle = -g_{J/\psi}\,\varepsilon^{*\mu}(\lambda)\,. \tag{2.13}$$

In the non-relativistic limit, $g_{J/\psi}$ is linked to $\psi_{J/\psi}(0)$ through the relation $g_{J/\psi} = 2^{3/2}N_c^{1/2}\,m_c^{1/2}\,\psi_{J/\psi}(0)$, which can be derived from (2.3). The calculation is much easier than its QCD counterpart, and the reduced QED amplitude is

$$\mathcal{A}_{\rm em} = e_c e_b^2\, e^3\, N_c \sqrt{\frac{m_c}{m_b}}\,\frac{\psi_{\eta_b}(0)\,\psi_{J/\psi}(0)}{m_c^2\,(m_b^2-m_c^2)}\,. \tag{2.14}$$

Substituting (2.7) and (2.14) into the formula

$$\Gamma[\eta_b \to J/\psi\gamma] = \frac{|\mathbf{k}|^3}{4\,\pi}\,|\mathcal{A}_{\rm str}+\mathcal{A}_{\rm em}|^2\,, \tag{2.15}$$

[1] In conformity with the sign convention adopted in (2.3), a minus sign is compulsory to put in here, which takes into account the Grassmann nature of the quark field operator.

we then obtain the desired partial width. Here $|\mathbf{k}| = (m_b^2 - m_c^2)/m_b$ is the photon momentum in the η_b rest frame. This formula already takes into account the sum over transverse polarizations of both J/ψ and γ.

For phenomenological purpose, it is instead more convenient to have an expression for the branching ratio, where $\psi_{\eta_b}(0)$ drops out:

$$\mathrm{Br}[\eta_b \to J/\psi\gamma] = \frac{8\, e_c^2\, \alpha\, \alpha_s^2}{3\,\pi} \frac{m_c\, \psi^2_{J/\psi}(0)}{m_b^2\,(m_b^2 - m_c^2)} \left| f\left(\frac{m_c^2}{m_b^2}\right) - g\left(\frac{m_c^2}{m_b^2}\right)\right|^2 , \tag{2.16}$$

where

$$g(u) = \frac{9\,\pi\, e_b^2\, \alpha}{2\,\alpha_s^2} \frac{1-u}{u} \tag{2.17}$$

encodes the QED fragmentation contribution. In deriving this, we have approximated the total width of η_b by its gluonic width:

$$\Gamma_{\rm tot}[\eta_b] \approx \Gamma[\eta_b \to gg] = \frac{8\pi\, \alpha_s^2}{3\, m_b^2}\, \psi^2_{\eta_b}(0)\,, \tag{2.18}$$

where the LO expression in α_s and v_b is used for simplicity.

Eq. (2.16) constitutes the key formula of this work. This equation conveys that, despite the adversity caused by the suppression α/α_s^2, the QED fragmentation contribution nevertheless enjoys the kinematic enhancement of m_b^2/m_c^2 relative to the QCD amplitude. For the physical masses of b and c, these two competing effects have comparable magnitudes. In the asymptotic regime as $m_b/m_c \gg 1$, the QED amplitude will eventually dominate over its QCD counterpart, because the enhancement factor m_b^2/m_c^2 of the former is much more eminent than the mild $\log(m_b^2/m_c^2)$ rising of the latter. At any rate, it is imperative to include the QED fragmentation contribution. Moreover, it is important to recognize that the interference between these two amplitudes is *destructive*. This is opposite to what occurs in the analogous decay process $\Upsilon \to \eta_c\gamma$ [21] and in the decay $\eta_b \to J/\psi\, J/\psi$ [13], where the interference is *constructive*.

3. Phenomenology

3.1 Observation potential of $\eta_b \to J/\psi\gamma$ at Tevatron and LHC

It is now the time to explore the phenomenological implication of (2.16). The input parameters are m_b, m_c, α, α_s and $\psi_{J/\psi}(0)$, all of which can be inferred from other independent sources. The wave function at the origin for J/ψ can be extracted from its electronic width:

$$\Gamma[J/\psi \to e^+e^-] = \frac{4\pi\, e_c^2\, \alpha^2}{m_c^2} \psi^2_{J/\psi}(0)\,, \tag{3.1}$$

where the LO formula in α_s and v_c^2 is used for simplicity. Using the measured dielectron width 5.55 keV [23], we obtain $\psi_{J/\psi}(0) = 0.205\ \mathrm{GeV}^{3/2}$ for $m_c = 1.5\,\mathrm{GeV}$. Taking $m_b = M_{\eta_b}/2 \approx 4.7\,\mathrm{GeV}$, $m_c = 1.5\,\mathrm{GeV}$, $\alpha = 1/137$ and $\alpha_s(m_b) = 0.22$, we then find

$$\mathrm{Br}[\eta_b \to J/\psi\,\gamma] = (1.5 \pm 0.8) \times 10^{-7}\,. \tag{3.2}$$

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The uncertainty is estimated by varying $\alpha_s(\mu)$ between 0.18 and 0.26 (which corresponds to slide the scale from $\mu = 2m_b$ to $2m_c$), as well as taking into account the errors in the measured $\Gamma_{e^+e^-}$ (of ± 0.14 keV). The destructive interference between electromagnetic and strong amplitudes has pronounced effect. For the central values of input parameters, omitting the QED contribution will result in a prediction of 3.5×10^{-7}, which is more than twice larger than the actual value. To develop a concrete perception, we enumerate the values of f and g evaluated at $u = m_c^2/m_b^2 = 0.10$ and $\alpha_s = 0.22$:

$$f = 3.5 - 2.4\,i\,, \qquad\qquad g = 2.1\,. \tag{3.3}$$

Clearly, Ref, Imf and g all have comparable magnitudes. As a result, the destructive interference effect is particularly important.

The numerical prediction presented in (3.2) is obtained by only using the tree-level NRQCD matching coefficients for the total η_b width and leptonic width of J/ψ. One may worry that this oversimplified procedure will induce some error because it is known that the next-to-leading perturbative corrections to both quantities are large. Let us assess their effects now. To the NLO accuracy in α_s, one needs to multiply eq. (2.18) by $1 + (53/2 - 31\,\pi^2/24 - 8\,n_f/9)\,\alpha_s(2m_b)/\pi$ (n_f stands for the number of active light flavors), as well as multiply eq. (3.1) by $(1 - 8\alpha_s(2m_c)/3\pi)^2$ [17]. Incorporating these corrections amounts to multiplying (3.2) by a factor [2]

$$\left(1 + \frac{10.2\,\alpha_s(2m_b)}{\pi}\right)^{-1} \left(1 - \frac{8\alpha_s(2m_c)}{3\pi}\right)^{-2} = 1.04\,, \tag{3.4}$$

where $n_f = 4$ has been taken. Since including the radiative corrections has negligible net impact, we will keep using (3.2) in the following phenomenological analysis.

It is enlightening to compare (3.2) with the NRQCD prediction to the branching ratio for η_b decays to double J/ψ [13]:

$$\mathrm{Br}[\eta_b \to J/\psi\, J/\psi] = 2.4^{+4.2}_{-1.9} \times 10^{-8}\,. \tag{3.5}$$

Notice that the branching ratio of our radiative decay process is almost one order-of-magnitude larger than that of this hadronic decay process! The reason can be traced as follows. The double J/ψ decay mode, though being a hadronic one, has maximally violated the helicity selection rule of pQCD. As a result, the branching ratio gets severely suppressed, $\propto \alpha_s^2\, v_c^{10} (m_c/m_b)^8$ [13]. In contrast, if we count $f \sim \mathcal{O}(1)$,[3] eq. (2.16) then implies that our radiative decay process admits a scaling behavior Br $\sim \alpha\, \alpha_s^2\, v_c^3\, (m_c/m_b)^4$, which is much more mildly suppressed by powers of $1/m_b^2$ and v_c relative to $\eta_b \to J/\psi J/\psi$, hence is more favorable even though it carries an extra factor of α.

Experimentally, J/ψ can be cleanly reconstructed by decays to lepton pairs. Multiplying (3.2) by the branching ratios of 12% for J/ψ decays to $\mu^+\mu^-$ and e^+e^-, we obtain

[2] It should be kept in mind that the relativistic correction to leptonic decay of J/ψ is also large. We have refrained from considering this complication.

[3] The slow rising $\ln(m_b^2/m_c^2)$ term in Ref is of no concern here for physical b and c masses. We also neglect the pure QED contribution for the lucidity of the argument.

$\mathrm{Br}[\eta_b \to J/\psi\gamma \to l^+l^-\gamma] = (0.8 - 2.8) \times 10^{-8}$. The total cross section for producing η_b at Tevatron has been estimated to be about 2.5 μb [12]. The production cross section for the $l^+l^-\gamma$ events is thus about $0.02 - 0.07$ pb. For the full Run 1 data of 100 pb^{-1}, we then obtain between 2 and 7 produced events. Because the kinematical cuts, as well as taking into account the acceptance and efficiency for detecting leptons, will further cut down this number, it seems not fruitful to assiduously seek the η_b through this radiative decay mode in Run 1 data sample.

Tevatron Run 2 aims to achieve an integrated luminosity of 8.5 fb^{-1} by 2009. Assuming equal $\sigma(p\overline{p} \to \eta_b + X)$ at $\sqrt{s} = 1.96$ and 1.8 TeV, we then estimate there are about 200-600 produced events. The product of acceptance and efficiency for detecting J/ψ decay to muon pair is estimated to be $\epsilon \approx 0.1$ [10]. It may sound reasonable to assume the corresponding factor for the electron pair also of the same magnitude. Multiplying the number of the produced events by ϵ, we expect between 20 and 60 observed events in the full Run 2 period. This is quite encouraging, but we should be cautious about the fact that the major QCD background events, the associated $J/\psi + \gamma$ production, could also be copious. At the Tevatron and LHC colliders, the dominant production mechanism for these events is through gg fusion [14 − 16]. A naive analysis indicates that the direct $J/\psi + \gamma$ production with an invariant mass near 9.4 GeV preponderates over that from the η_b decays.

It is important to keep in mind that the J/ψ stemming from the radiative η_b decay must be transversely polarized. This characteristic may be used to discriminate the signal events from the background events, because J/ψ from the latter processes can also be longitudinally polarized. Furthermore, the detailed kinematical distributions of background processes need to be thoroughly studied, in order to guide experimentalists to choose the optimal kinematical cuts to oppress as many background as possible, and in the meanwhile without significantly sacrificing the signal events. This is somewhat beyond the scope of this work and needs further independent studies.

The forthcoming LHC experiments will greatly increase the number of the produced $\eta_b \to l^+l^-\gamma$ events. To assess the discovery potential of this mode at LHC, we need first know the inclusive production rate for η_b. There are rough estimates for the $\chi_{b0,2}$ cross sections at LHC, which are about 6 times larger than the corresponding ones for producing them at Tevatron [24]. Assuming the same scaling also holds for η_b, we then obtain the cross section for η_b at LHC to be about 15 μb, subsequently the production cross section for the $l^+l^-\gamma$ events to be about 0.1-0.4 pb. For 300 fb^{-1} data, which is expected to be collected in one year run at LHC design luminosity, the number of produced events may reach about $3 \times 10^4 - 1 \times 10^5$. Multiplying the number of the produced events by ϵ, we expect between 3×10^3 and 1×10^4 observed events per year.

Based on this analysis, we are tempted to conclude that, the chance of observing η_b at LHC through this mode is very promising. With such a large amount of signal events, it is possible to measure the leptonic angular distribution to pin down the polarization of J/ψ. As has been stressed, in order to effectively select the signal events out of the abundant background events, one needs to develop a thorough understanding towards the QCD background.

3.2 Radiative decay of η_b (η_c) into ϕ

When coping with light mesons in hard exclusive processes, the most natural treatment for them is using light-cone wave functions. Indeed, heavy quarkonium radiative decays to light mesons, such as $\Upsilon \to f_2(1270)\gamma$ [25, 20] (ref. [20] has heavily exploited the EFT machinery, i.e. NRQCD combined with SCET), $J/\psi(\Upsilon) \to \eta(\eta')\gamma$ [26, 27], have been studied along this line. On the other hand, the constituent quark model, which treats the light meson as a non-relativistic bound state, is also frequently invoked as an alternative method for a quick order-of-magnitude estimate. Various radiative decay processes, e.g. J/ψ decays into light pseudo-scalar and P-wave mesons, as well as χ_{cJ} into light vector mesons, have already been studied in this context [22, 28].

We may apply the same strategy to analyze the radiative decay $\eta_b(\eta_c) \to V\gamma$. We take $\eta_b \to \phi\gamma$ as a representative. By regarding ϕ as a strangeonium, we can directly use (2.16), only with some trivial changes of input parameters. We take the $m_s = M_\phi/2 \approx 0.5\,\text{GeV}$. The wave function at the origin of ϕ, $\psi_\phi(0)$, can be extracted through (3.1) from its measured electronic width of 1.27 ± 0.04 keV [23]. Taking $m_b = 4.7\,\text{GeV}$, varying the strong coupling constant between $\alpha_s(2m_b) = 0.18$ and $\alpha_s(2m_s) = 0.51$, and including the experimental uncertainty in $\Gamma_{e^+e^-}$, we obtain

$$\text{Br}[\eta_b \to \phi\gamma] = (0.3 - 6.9) \times 10^{-8}\,. \tag{3.6}$$

The QED contribution dominates in this case due to the larger ratio of m_b to m_s. To see this concretely, we list the values of f and g evaluated at $u = m_s^2/m_b^2 \approx 0.01$ and $\alpha_s = 0.22$:

$$f = 4.9 - 1.5\,i\,, \qquad\qquad g = 23.5\,. \tag{3.7}$$

The dominance of g over $|f|$ is apparent. Neglecting QED contribution for this α_s value will decrease the branching fraction by one order of magnitude. For the lack of clean signature to efficiently tag ϕ, such a rare decay mode will be rather difficult to observe in hadron collision experiments such as LHC. It may be interesting to compare this radiative decay channel with the following hadronic mode [13]:

$$\text{Br}[\eta_b \to \phi\phi] = (0.9 - 1.4) \times 10^{-9}\,. \tag{3.8}$$

This double ϕ mode has even smaller branching ratio, and seems hopeless to be seen experimentally.

One can proceed to consider the analogous decay process $\eta_c \to \phi\gamma$. Parallel to the preceding analysis, taking $m_c = 1.5\,\text{GeV}$, and varying $\alpha_s(\mu)$ from $\alpha_s(2m_c) = 0.26$ to $\alpha_s(2m_s) = 0.51$, we obtain

$$\text{Br}[\eta_c \to \phi\gamma] = (2.1 - 8.6) \times 10^{-7}\,. \tag{3.9}$$

The destructive interference pattern is similar to that in $\eta_b \to J/\psi\gamma$. This decay mode is still too much suppressed to be observed in the functioning charmonium factory like BES II, perhaps also in the forthcoming BES III experiment. It seems also rather difficult to observe this decay mode in the current and future hadron collision facilities.

4. Summary

The motif of this work is to suggest one viable way to ferret out η_b in the functioning and forthcoming hadron collider facilities, that is, through its radiative decay into J/ψ. This decay mode owns the advantage that both J/ψ and photon can be tagged cleanly. The presence of J/ψ is particularly helpful to reduce the combinatorial background. In this regard, this mode is practically much more useful than the purely electromagnetic decay $\eta_b \to \gamma\gamma$.

By an explicit pQCD calculation based on NRQCD approach, we infer the branching ratio of this process to be of order 10^{-7}. Although the absolute value of this ratio is small, it is already larger than that of the hadronic decay mode $\eta_b \to J/\psi J/\psi$, which was previously thought of as a golden mode for searching η_b. Our analysis indicates that the chance of observing η_b through this decay channel, followed by J/ψ decay to a lepton pair, with a corresponding branching ratio about 10^{-8}, seems still open in Tevatron Run 2, and is quite promising in the forthcoming LHC experiment. However, one should bear in mind that the major QCD background events are expected to greatly outnumber the desired signal events. A thorough study of the associated $J/\psi + \gamma$ production is welcome, in order to help experimentalists to impose optimal kinematical cuts to singlet out the signal events from the abundant background events. The transverse polarization of J/ψ in the signal events should be employed to effectively veto the background events.

While this paper is being written, we are informed of the same $\eta_b \to J/\psi\gamma$ process being also considered by Gao, Zhang and Chao [30]. These authors evaluated loop integrals numerically. It has been checked that once the same input parameters are assumed, our numerical prediction for the branching ratio is compatible with theirs. These authors have also studied various other radiative decay channels of bottomonia to charmonia.

Acknowledgments

The work of G. H., C. F. Q. and P. S. was supported in part by the Natural Science Foundation of China and by the Scientific Research Fund of GUCAS (NO. 055101BM03). Y. J. is supported in part by NSFC under Grant No. 10605031 (effective since January 2007).

A. Deriving analytical expression for f

In this appendix we illustrate how to reduce the one-loop four-point function in (2.8) to the sum of much simpler two- and three-point scalar integrals. We start with the dimensionless integral f:

$$f\left(\frac{m_c^2}{m_b^2}\right) = \frac{8}{i\pi^2}\int d^4q \frac{(k\cdot Q)\,q^2 - (k\cdot q)\,(Q\cdot q)}{(q+Q)^2\,(q-Q)^2\,(q^2+2k\cdot q-P^2)\,(q^2-2k\cdot q-P^2)}\,, \tag{A.1}$$

where $Q^2 = 4m_b^2$ and $P^2 = 4m_c^2$. The $+i\varepsilon$ prescription in the propagators is implicitly implied.

First note the second term in the integrand can be simplified by using the identity of fractional sum:

$$\begin{aligned}
&\int d^4q \frac{(k\cdot q)\,(Q\cdot q)}{(q+Q)^2\,(q-Q)^2\,(q^2+2k\cdot q-P^2)\,(q^2-2k\cdot q-P^2)} \\
&\quad = \frac{1}{16}\int d^4q \left[\frac{1}{(q-Q)^2}-\frac{1}{(q+Q)^2}\right]\left[\frac{1}{q^2-2k\cdot q-P^2}-\frac{1}{q^2+2k\cdot q-P^2}\right] \\
&\quad = \frac{1}{8}\int d^4q\, \frac{1}{(q-Q)^2}\left[\frac{1}{q^2-2k\cdot q-P^2}-\frac{1}{q^2+2k\cdot q-P^2}\right],
\end{aligned} \tag{A.2}$$

which is nothing but the scalar 2-point functions, thus can be trivially worked out.

Disentangling the first term in (A.1) needs slightly more labor. Since the integrand is an even function of q, we are free to add terms linear in q in the numerator, without influencing the result:

$$\begin{aligned}
&\int d^4q\, \frac{q^2}{(q+Q)^2\,(q-Q)^2\,(q^2+2k\cdot q-P^2)\,(q^2-2k\cdot q-P^2)} \\
&\quad = \frac{1}{Q^2+P^2}\int d^4q\, \frac{(Q^2+P^2)\,q^2+2Q^2\,k\cdot q+2P^2\,Q\cdot q+Q^2P^2-Q^2P^2}{(q+Q)^2\,(q-Q)^2\,(q^2+2k\cdot q-P^2)\,(q^2-2k\cdot q-P^2)} \\
&\quad = \frac{Q^2}{Q^2+P^2}\int d^4q\, \frac{1}{(q+Q)^2\,(q-Q)^2\,(q^2-2k\cdot q-P^2)} \\
&\qquad +\frac{P^2}{Q^2+P^2}\int d^4q\, \frac{1}{(q-Q)^2\,(q^2+2k\cdot q-P^2)(q^2-2k\cdot q-P^2)},
\end{aligned} \tag{A.3}$$

which consists of two independent 3-point scalar integrals. They can also be worked out in closed form, following the method outlined in ref. [29].

With the aid of (A.2) and (A.3), we can decompose the original f into three pieces:

$$f(u) = f_1(u)+f_2(u)+f_3(u), \tag{A.4}$$

where $u\equiv P^2/Q^2 = m_c^2/m_b^2$, and

$$\begin{aligned}
f_1(u) &= \frac{i}{\pi^2}\int d^4q\, \frac{1}{(q-Q)^2}\left[\frac{1}{q^2-2k\cdot q-P^2}-\frac{1}{q^2+2k\cdot q-P^2}\right] \\
&= \frac{2\,(1-u)}{2-u}\,\ln\left[\frac{u}{2(1-u)}\right]+2\pi\, i\left(\frac{1-u}{2-u}\right),
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
f_2(u) &= \frac{8\,Q^2\,k\cdot Q}{i\pi^2(Q^2+P^2)}\int d^4q\, \frac{1}{(q+Q)^2\,(q-Q)^2\,(q^2-2k\cdot q-P^2)} \\
&= -\frac{2}{1+u}\left\{\ln^2 2+\frac{1}{2}\ln^2 u+\ln[2-u]\,\ln\left[\frac{u}{2(1-u)}\right]+\ln u\,\ln\left[\frac{2}{1-u}\right]\right. \\
&\quad \left.+\,2\,\mathrm{Li}_2[-u]+\mathrm{Li}_2\left[\frac{u-1}{2u}\right]+2\,\mathrm{Li}_2\left[\frac{u}{2}\right]-\mathrm{Li}_2\left[\frac{u^2-u}{2}\right]\right\}-\frac{2\pi\, i}{1+u}\,\ln[2-u],
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
f_3(u) &= \frac{8\,P^2\,k\cdot Q}{i\pi^2(Q^2+P^2)}\int d^4q\, \frac{1}{(q-Q)^2\,(q^2+2k\cdot q-P^2)(q^2-2k\cdot q-P^2)} \\
&= \frac{2\,u}{1+u}\left\{\ln\left[\frac{u}{2-u}\right]\ln\left[\frac{u}{2(1-u)}\right]+\mathrm{Li}_2\left[2-\frac{2}{u}\right]\right\}+2\pi i\left(\frac{u}{1+u}\right)\ln\left[\frac{u}{2-u}\right].
\end{aligned} \tag{A.7}$$

One then readily reproduces the analytic results shown in (2.9) and (2.10).

References

[1] For a comprehensive review on various aspects of η_b, see QUARKONIUM WORKING GROUP collaboration, N. Brambilla et al., *Heavy quarkonium physics*, hep-ph/0412158.

[2] D. Ebert, R.N. Faustov and V.O. Galkin, *Properties of heavy quarkonia and B_c mesons in the relativistic quark model*, *Phys. Rev.* **D 67** (2003) 014027 [hep-ph/0210381].

[3] S. Recksiegel and Y. Sumino, *Fine and hyperfine splittings of charmonium and bottomonium: an improved perturbative QCD approach*, *Phys. Lett.* **B 578** (2004) 369 [hep-ph/0305178].

[4] B.A. Kniehl, A.A. Penin, A. Pineda, V.A. Smirnov and M. Steinhauser, *$M(\eta_b)$ and α_s from non-relativistic renormalization group*, *Phys. Rev. Lett.* **92** (2004) 242001 [hep-ph/0312086].

[5] A. Gray et al., *The Υ spectrum and m_b from full lattice QCD*, *Phys. Rev.* **D 72** (2005) 094507 [hep-lat/0507013].

[6] ALEPH collaboration, A. Heister et al., *Search for $\gamma\gamma \to \eta_b$ in e^+e^- collisions at LEP2*, *Phys. Lett.* **B 530** (2002) 56 [hep-ex/0202011].

[7] L3 collaboration, M. Levtchenko, *Heavy-quark pseudoscalar formation in two-photon collisions at LEP*, *Nucl. Phys.* **126** *(Proc. Suppl.)* (2004) 260.

[8] DELPHI collaboration, J. Abdallah, *Search for η_b in two-photon collisions at LEP II with the DELPHI detector*, *Phys. Lett.* **B 634** (2006) 340 [hep-ex/0601042].

[9] CLEO collaboration, M. Artuso et al., *Photon transitions in $\Upsilon(2S)$ and $\Upsilon(3S)$ decays*, *Phys. Rev. Lett.* **94** (2005) 032001 [hep-ex/0411068].

[10] E. Braaten, S. Fleming and A.K. Leibovich, *NRQCD analysis of bottomonium production at the Tevatron*, *Phys. Rev.* **D 63** (2001) 094006 [hep-ph/0008091].

[11] CDF collaboration, J. Tseng, *Search for η_b at CDF*, Presented at 5th International Conference on Quark Confinement and the Hadron Spectrum, Gargnano, Brescia, Italy, 10-14 Sep 2002, FERMILAB-CONF-02-348-E.

[12] F. Maltoni and A.D. Polosa, *Observation potential for η_b at the Tevatron*, *Phys. Rev.* **D 70** (2004) 054014 [hep-ph/0405082].

[13] Y. Jia, *Which hadronic decay modes are good for η_b searching: double J/ψ or something else?*, hep-ph/0611130.

[14] D.P. Roy and K. Sridhar, *$J/\psi + \gamma$ production at the Tevatron energy*, *Phys. Lett.* **B 341** (1995) 413 [hep-ph/9407390].

[15] C.S. Kim, J. Lee and H.S. Song, *Color-octet contributions in the associate $J/\psi + \gamma$ hadroproduction*, *Phys. Rev.* **D 55** (1997) 5429 [hep-ph/9610294].

[16] P. Mathews, K. Sridhar and R. Basu, *$J/\psi + \gamma$ production at the LHC*, *Phys. Rev.* **D 60** (1999) 014009 [hep-ph/9901276].

[17] G.T. Bodwin, E. Braaten and G.P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, *Phys. Rev.* **D 51** (1995) 1125 [hep-ph/9407339].

[18] N. Brambilla, D. Eiras, A. Pineda, J. Soto and A. Vairo, *Inclusive decays of heavy quarkonium to light particles*, *Phys. Rev.* **D 67** (2003) 034018 [hep-ph/0208019].

[19] N. Brambilla, Y. Jia and A. Vairo, *Model-independent study of magnetic dipole transitions in quarkonium*, *Phys. Rev.* **D 73** (2006) 054005 [hep-ph/0512369].

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[20] S. Fleming, C. Lee and A.K. Leibovich, *Exclusive radiative decays of Υ in SCET*, *Phys. Rev.* **D 71** (2005) 074002 [hep-ph/0411180].

[21] B. Guberina and J.H. Kuhn, *Complete evaluation of $\Upsilon \to \gamma\eta_c$ in the quarkonium model*, *Lett. Nuovo Cim.* **32** (1981) 295.

[22] J.G. Korner, J.H. Kuhn, M. Krammer and H. Schneider, *Zweig forbidden radiative orthoquarkonium decays in perturbative QCD*, *Nucl. Phys.* **B 229** (1983) 115.

[23] W.M. Yao et al. [Particle Data Group], *Review of particle physics*, *J. Phys.* **G 33** (2006) 1.

[24] V.V. Braguta, A.K. Likhoded and A.V. Luchinsky, *Observation potential for χ_b at the Tevatron and LHC*, *Phys. Rev.* **D 72** (2005) 094018 [hep-ph/0506009].

[25] J.P. Ma, *A QCD-analysis for radiative decays of upsilon into $f_2(1270)$*, *Nucl. Phys.* **B 605** (2001) 625 [hep-ph/0103237].

[26] Y.-D. Yang, *Radiative decays $J/\psi \to \eta^{(\prime)}\gamma$ in perturbative QCD*, hep-ph/0404018.

[27] G. Li, T. Li, X.-Q. Li, W.-G. Ma and S.-M. Zhao, *Revisiting the OZI-forbidden radiative decays of orthoquarkonia*, *Nucl. Phys.* **B 727** (2005) 301 [hep-ph/0505158].

[28] Y.-J. Gao, Y.-J. Zhang and K.-T. Chao, *Radiative decays of charmonium into light mesons*, *Chin. Phys. Lett.* **23** (2006) 2376–2378 [hep-ph/0607278].

[29] G. 't Hooft and M.J.G. Veltman, *Scalar one loop integrals*, *Nucl. Phys.* **B 153** (1979) 365.

[30] Y.-J. Gao, Y.-J. Zhang and K.-T. Chao, *Radiative decays of bottomonia into charmonia and light mesons*, hep-ph/0701009.